

AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

- 1 1. (Currently amended) A method for using a computer system to solve a
2 global inequality constrained optimization problem specified by a function f and a
3 set of inequality constraints $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$), wherein f and p_i are scalar
4 functions of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the method comprising:
5 receiving a representation of the function f and the set of inequality
6 constraints at the computer system;
7 storing the representation in a memory within the computer system;
8 performing an interval inequality constrained global optimization process
9 to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$
10 subject to the set of inequality constraints;
11 wherein performing the interval global optimization process involves,
12 applying term consistency to the set of inequality
13 constraints over a ~~sub-box~~sub-box \mathbf{X} , and
14 excluding any portion of the ~~sub-box~~sub-box \mathbf{X} that is
15 proved to be in violation of at least one member of the set of
16 inequality constraints; and
17 recording the guaranteed bounds in the computer system memory.
- 1 2. (Currently amended) The method of claim 1, further comprising:

2 linearizing the set of inequality constraints to produce a set of linear
3 inequality constraints with interval coefficients that enclose the nonlinear
4 constraints;
5 preconditioning the set of linear inequality constraints through additive
6 linear combinations to produce a preconditioned set of linear inequality
7 constraints;
8 applying term consistency to the set of preconditioned linear inequality
9 constraints over the ~~sub-box~~sub-box \mathbf{X} , and
10 excluding any portion of the ~~sub-box~~sub-box \mathbf{X} that violates any member
11 of the set of preconditioned linear inequality constraints.

1 3. (Original) The method of claim 2, further comprising:
2 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
3 point \mathbf{x} wherein $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$); and
4 including $f(\mathbf{x}) \leq f_bar$ in the set of inequality constraints prior to
5 linearizing the set of inequality constraints.

1 4. (Original) The method of claim 2, further comprising removing from
2 consideration any inequality constraints that are not violated by more than a
3 specified amount for purposes of applying term consistency prior to linearizing
4 the set of inequality constraints.

1 5. (Currently amended) The method of claim 1, wherein performing the
2 interval global optimization process involves:
3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 point \mathbf{x} ;
5 removing from consideration any ~~sub-box~~sub-box for which $f(\mathbf{x}) > f_bar$;

6 applying term consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the
7 ~~sub~~~~box~~sub-box \mathbf{X} ; and
8 excluding any portion of the ~~sub~~~~box~~sub-box \mathbf{X} that violates the f_bar
9 inequality.

1 6. (Currently amended) The method of claim 1, wherein if the ~~sub~~~~box~~sub-
2 ~~box~~ \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval
3 global optimization process involves:
4 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
5 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
6 removing from consideration any ~~sub~~~~box~~sub-box for which $\mathbf{g}(\mathbf{x})$ is
7 bounded away from zero, thereby indicating that the ~~sub~~~~box~~sub-box does not
8 include an extremum of $f(\mathbf{x})$; and
9 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$
10 over the ~~sub~~~~box~~sub-box \mathbf{X} ; and
11 excluding any portion of the ~~sub~~~~box~~sub-box \mathbf{X} that violates any
12 component of $\mathbf{g}(\mathbf{x})=\mathbf{0}$.

1 7. (Currently amended) The method of claim 1, wherein if the ~~sub~~~~box~~sub-
2 ~~box~~ \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval
3 global optimization process involves:
4 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
5 function $f(\mathbf{x})$;
6 removing from consideration any ~~sub~~~~box~~sub-box for which $H_{ii}(\mathbf{x})$ a
7 diagonal element of the Hessian over the ~~sub~~~~box~~sub-box \mathbf{X} is always negative,
8 indicating that the function f is not convex over the ~~sub~~~~box~~sub-box \mathbf{X} and
9 consequently does not contain a global minimum within the ~~sub~~~~box~~sub-box \mathbf{X} ;

10 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
11 ~~sub-box~~sub-box \mathbf{X} ; and
12 excluding any portion of the ~~sub-box~~sub-box \mathbf{X} that violates a Hessian
13 inequality.

1 8. (Currently amended) The method of claim 1, wherein if the ~~sub-box~~sub-
2 box \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$), performing the interval
3 global optimization process involves:
4 performing the Newton method, wherein performing the Newton method
5 involves,
6 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient of the
7 function f evaluated with respect to a point \mathbf{x} over the ~~sub-box~~sub-
8 box \mathbf{X} ,
9 computing an approximate inverse \mathbf{B} of the center of
10 $\mathbf{J}(\mathbf{x}, \mathbf{X})$,
11 using the approximate inverse \mathbf{B} to analytically determine
12 the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
13 and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$);
14 applying term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for
15 each variable x_i ($i=1, \dots, n$) over the ~~sub-box~~sub-box \mathbf{X} ; and
16 excluding any portion of the ~~sub-box~~sub-box \mathbf{X} that violates a component.

1 9. (Currently amended) The method of claim 1, wherein applying term
2 consistency involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a term, $g(x'_j)$, thereby producing a modified equation $g(x'_j) = h(\mathbf{x})$,
5 wherein the term $g(x'_j)$ can be analytically inverted to produce an inverse function
6 $g^{-1}(\mathbf{y})$;

7 substituting the ~~sub-box~~ X into the modified equation to produce
 8 the equation $g(X'_j) = h(X)$;
 9 solving for $X'_j = g^{-1}(h(X))$; and
 10 intersecting X'_j with the j -th element of the ~~sub-box~~ X to produce a
 11 new ~~sub-box~~ X^+ ;
 12 wherein the new ~~sub-box~~ X^+ contains all solutions of the equation
 13 within the ~~sub-box~~ X, and wherein the size of the new ~~sub-box~~ X^+
 14 is less than or equal to the size of the ~~sub-box~~ X.

1 10. (Original) The method of claim 1, further comprising performing the
 2 Newton method on the John conditions.

1 11. (Currently amended) A computer-readable storage medium storing
 2 instructions that when executed by a computer cause the computer to perform a
 3 method for using a computer system to solve a global inequality constrained
 4 optimization problem specified by a function f and a set of inequality constraints
 5 $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$), wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$,
 6 the method comprising:
 7 receiving a representation of the function f and the set of inequality
 8 constraints at the computer system;
 9 storing the representation in a memory within the computer system;
 10 performing an interval inequality constrained global optimization process
 11 to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$
 12 subject to the set of inequality constraints;
 13 wherein performing the interval global optimization process involves,
 14 applying term consistency to the set of inequality
 15 constraints over a ~~sub-box~~ X, and

16 | excluding any portion of the ~~sub-box~~sub-box X that is
17 | proved to be in violation of at least one member of the set of
18 | inequality constraints; and
19 | recording the guaranteed bounds in the computer system memory.

1 12. (Currently amended) The computer-readable storage medium of claim
2 | 11, wherein the method further comprises:
3 | linearizing the set of inequality constraints to produce a set of linear
4 | inequality constraints with interval coefficients that enclose the nonlinear
5 | constraints;
6 | preconditioning the set of linear inequality constraints through additive
7 | linear combinations to produce a preconditioned set of linear inequality
8 | constraints;
9 | applying term consistency to the set of preconditioned linear inequality
10 | constraints over the ~~sub-box~~sub-box X, and
11 | excluding any portion of the ~~sub-box~~sub-box X that violates any member
12 | of the set of preconditioned linear inequality constraints.

1 13. (Original) The computer-readable storage medium of claim 12,
2 | wherein the method further comprises:
3 | keeping track of a least upper bound f_bar of the function $f(x)$ at a feasible
4 | point x wherein $p_i(x) \leq 0$ ($i=1, \dots, m$); and
5 | including $f(x) \leq f_bar$ in the set of inequality constraints prior to
6 | linearizing the set of inequality constraints.

1 14. (Original) The computer-readable storage medium of claim 12,
2 | wherein the method further comprises removing from consideration any inequality

3 constraints that are not violated by more than a specified amount for purposes of
4 applying term consistency prior to linearizing the set of inequality constraints.

1 15. (Currently amended) The computer-readable storage medium of claim
2 11, wherein performing the interval global optimization process involves:

3 keeping track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 point \mathbf{x} ;

5 removing from consideration any ~~sub-box~~ sub-box for which $f(\mathbf{x}) > f_bar$;

6 applying term consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the

7 ~~sub-box~~ sub-box \mathbf{X} ; and

8 excluding any portion of the ~~sub-box~~ sub-box \mathbf{X} that violates the f_bar
9 inequality.

1 16. (Currently amended) The computer-readable storage medium of claim
2 11, wherein if the ~~sub-box~~ sub-box \mathbf{X} is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$),
3 performing the interval global optimization process involves:

4 determining a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes
5 components $g_i(\mathbf{x})$ ($i=1, \dots, n$);

6 removing from consideration any ~~sub-box~~ sub-box for which $\mathbf{g}(\mathbf{x})$ is
7 bounded away from zero, thereby indicating that the ~~sub-box~~ sub-box does not
8 include an extremum of $f(\mathbf{x})$; and

9 applying term consistency to each component $g_i(\mathbf{x})=0$ ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=0$
10 over the ~~sub-box~~ sub-box \mathbf{X} ; and

11 excluding any portion of the ~~sub-box~~ sub-box \mathbf{X} that violates any
12 component of $\mathbf{g}(\mathbf{x})=0$.

1 17. (Currently amended) The computer-readable storage medium of claim
2 | 11, wherein if the ~~sub~~box~~sub-box~~ **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$),
3 performing the interval global optimization process involves:
4 determining diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the Hessian of the
5 function $f(\mathbf{x})$;
6 | removing from consideration any ~~sub~~box~~sub-box~~ for which $H_{ii}(\mathbf{x})$ a
7 diagonal element of the Hessian over the ~~sub~~box~~sub-box~~ **X** is always negative,
8 indicating that the function f is not convex over the ~~sub~~box~~sub-box~~ **X** and
9 consequently does not contain a global minimum within the ~~sub~~box~~sub-box~~ **X**;
10 applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ ($i=1, \dots, n$) over the
11 ~~sub~~box~~sub-box~~ **X**; and
12 excluding any portion of the ~~sub~~box~~sub-box~~ **X** that violates a Hessian
13 inequality.

1 18. (Currently amended) The computer-readable storage medium of claim
2 | 11, wherein if the ~~sub~~box~~sub-box~~ **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$),
3 performing the interval global optimization process involves:
4 performing the Newton method, wherein performing the Newton method
5 involves,
6 computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient of the
7 | function f evaluated with respect to a point \mathbf{x} over the ~~sub~~box~~sub-~~
8 | box **X**,
9 computing an approximate inverse \mathbf{B} of the center of
10 $\mathbf{J}(\mathbf{x}, \mathbf{X})$,
11 using the approximate inverse \mathbf{B} to analytically determine
12 the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
13 and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$);

14 applying term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ ($i=1, \dots, n$) for
15 each variable x_i ($i=1, \dots, n$) over the ~~sub-box~~ sub-box \mathbf{X} ; and
16 excluding any portion of the ~~sub-box~~ sub-box \mathbf{X} that violates a component.

1 19. (Currently amended) The computer-readable storage medium of claim
2 11, wherein applying term consistency involves:
3 symbolically manipulating an equation within the computer system to
4 solve for a term, $g(x'_j)$, thereby producing a modified equation $g(x'_j) = h(\mathbf{x})$,
5 wherein the term $g(x'_j)$ can be analytically inverted to produce an inverse function
6 $g^{-1}(\mathbf{y})$;
7 substituting the ~~sub-box~~ sub-box \mathbf{X} into the modified equation to produce
8 the equation $g(\mathbf{X}'_j) = h(\mathbf{X})$;
9 solving for $\mathbf{X}'_j = g^{-1}(h(\mathbf{X}))$; and
10 intersecting \mathbf{X}'_j with the j -th element of the ~~sub-box~~ sub-box \mathbf{X} to produce a
11 new ~~sub-box~~ sub-box \mathbf{X}^+ ;
12 wherein the new ~~sub-box~~ sub-box \mathbf{X}^+ contains all solutions of the equation
13 within the ~~sub-box~~ sub-box \mathbf{X} , and wherein the size of the new ~~sub-box~~ sub-box \mathbf{X}^+
14 is less than or equal to the size of the ~~sub-box~~ sub-box \mathbf{X} .

1 20. (Original) The computer-readable storage medium of claim 11,
2 wherein the method further comprises performing the Newton method on the John
3 conditions.

1 21. (Currently amended) An apparatus for using a computer system to
2 solve a global inequality constrained optimization problem specified by a function
3 f and a set of inequality constraints $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$), wherein f is a scalar
4 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$, the apparatus comprising:

5 a receiving mechanism that is configured to receive a representation of the
6 function f and the set of inequality constraints at the computer system;
7 a memory within the computer system for storing the representation;
8 a global optimizer that is configured to perform an interval inequality
9 constrained global optimization process to compute guaranteed bounds on a
10 globally minimum value of the function $f(\mathbf{x})$ subject to the set of inequality
11 constraints;
12 a term consistency mechanism within the global optimizer that is
13 configured to,
14 apply term consistency to the set of inequality constraints
15 over a ~~sub-box~~sub-box \mathbf{X} , and to
16 exclude any portion of the ~~sub-box~~sub-box \mathbf{X} that is proved
17 to be in violation of at least one member of the set of inequality
18 constraints; and
19 a recording mechanism that is configured record the guaranteed bounds in
20 the computer system memory.

1 22. (Currently amended) The apparatus of claim 21, further comprising:
2 a linearizing mechanism that is configured to linearize the set of inequality
3 constraints to produce a set of linear inequality constraints with interval
4 coefficients that enclose the nonlinear constraints; and
5 a preconditioning mechanism that is configured to precondition the set of
6 linear inequality constraints through additive linear combinations to produce a
7 preconditioned set of linear inequality constraints;
8 wherein the term consistency mechanism is configured to,
9 apply term consistency to the set of preconditioned linear
10 inequality constraints over the ~~sub-box~~sub-box \mathbf{X} , and to

11 | exclude any portion of the ~~sub~~~~box~~sub-box **X** that violates
12 | any member of the set of preconditioned linear inequality
13 | constraints.

1 23. (Original) The apparatus of claim 22, wherein the global optimizer is
2 | configured to:

3 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$ at a feasible
4 | point \mathbf{x} wherein $p_i(\mathbf{x}) \leq 0$ ($i=1, \dots, m$); and to

5 include $f(\mathbf{x}) \leq f_bar$ in the set of inequality constraints prior to linearizing
6 | the set of inequality constraints.

1 24. (Original) The apparatus of claim 22, wherein the term consistency
2 | mechanism is configured to remove from consideration any inequality constraints
3 | that are not violated by more than a specified amount for purposes of applying
4 | term consistency prior to linearizing the set of inequality constraints.

1 25. (Currently amended) The apparatus of claim 21,
2 | wherein the global optimizer is configured to,

3 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$
4 | at a feasible point \mathbf{x} , and to

5 | remove from consideration any ~~sub~~~~box~~sub-box for which
6 | $f(\mathbf{x}) > f_bar$;

7 wherein the term consistency mechanism is configured to,
8 | apply term consistency to the f_bar

9 | inequality $f(\mathbf{x}) \leq f_bar$ over the ~~sub~~~~box~~sub-box **X**,
10 | and to

11 | exclude any portion of the ~~sub~~~~box~~sub-box **X**
12 | that violates the f_bar inequality.

1 26. (Currently amended) The apparatus of claim 21, wherein if the
2 | ~~sub-box~~ sub-box **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$):
3 the global optimizer is configured to,
4 determine a gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$
5 includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$), and to
6 | remove from consideration any ~~sub-box~~ sub-box for which
7 $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the
8 | ~~sub-box~~ sub-box does not include an extremum of $f(\mathbf{x})$; and
9 the term consistency mechanism is configured to,
10 apply term consistency to each component $g_i(\mathbf{x})=0$
11 | ($i=1, \dots, n$) of $\mathbf{g}(\mathbf{x})=0$ over the ~~sub-box~~ sub-box **X**, and to
12 | exclude any portion of the ~~sub-box~~ sub-box **X** that violates
13 any component of $\mathbf{g}(\mathbf{x})=0$.

1 27. (Currently amended) The apparatus of claim 21, wherein if the
2 | ~~sub-box~~ sub-box **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$):
3 the global optimizer is configured to,
4 determine diagonal elements $H_{ii}(\mathbf{x})$ ($i=1, \dots, n$) of the
5 Hessian of the function $f(\mathbf{x})$, and to
6 | remove from consideration any ~~sub-box~~ sub-box for which
7 | $H_{ii}(\mathbf{x})$ a diagonal element of the Hessian over the ~~sub-box~~ sub-box **X**
8 is always negative, indicating that the function f is not convex over
9 | the ~~sub-box~~ sub-box **X** and consequently does not contain a global
10 | minimum within the ~~sub-box~~ sub-box **X**; and
11 the term consistency mechanism is configured to,
12 apply term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$
13 | ($i=1, \dots, n$) over the ~~sub-box~~ sub-box **X**, and to

14 | exclude any portion of the ~~sub-box~~sub-box **X** that violates a
15 | Hessian inequality.

1 28. (Currently amended) The apparatus of claim 21, wherein if the
2 | ~~sub-box~~sub-box **X** is strictly feasible ($p_i(\mathbf{X}) < 0$ for all $i=1, \dots, n$):
3 | the global optimizer is configured to perform the Newton method, wherein
4 | performing the Newton method involves,
5 | computing the Jacobian $\mathbf{J}(\mathbf{x}, \mathbf{X})$ of the gradient of the
6 | function f evaluated with respect to a point \mathbf{x} over the ~~sub-box~~sub-
7 | box **X**,
8 | computing an approximate inverse **B** of the center of
9 | $\mathbf{J}(\mathbf{x}, \mathbf{X})$, and
10 | using the approximate inverse **B** to analytically determine
11 | the system $\mathbf{B}\mathbf{g}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$,
12 | and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ ($i=1, \dots, n$); and
13 | the term consistency mechanism is configured to,
14 | apply term consistency to each component $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$
15 | ($i=1, \dots, n$) for each variable x_i ($i=1, \dots, n$) over the ~~sub-box~~sub-box
16 | **X**, and to
17 | exclude any portion of the ~~sub-box~~sub-box **X** that violates a
18 | component.

1 29. (Currently amended) The apparatus of claim 21, wherein the term
2 | consistency mechanism is configured to:
3 | symbolically manipulate an equation within the computer system to solve
4 | for a term, $g(x'_j)$, thereby producing a modified equation $g(x'_j) = h(\mathbf{x})$, wherein
5 | the term $g(x'_j)$ can be analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

6 substitute the ~~subbox~~sub-box \mathbf{X} into the modified equation to produce the
7 equation $g(\mathbf{X}'_j) = h(\mathbf{X})$;
8 solve for $\mathbf{X}'_j = g^{-1}(h(\mathbf{X}))$; and
9 intersect \mathbf{X}'_j with the j -th element of the ~~subbox~~sub-box \mathbf{X} to produce a
10 new ~~subbox~~sub-box \mathbf{X}^+ ;
11 wherein the new ~~subbox~~sub-box \mathbf{X}^+ contains all solutions of the equation
12 within the ~~subbox~~sub-box \mathbf{X} , and wherein the size of the new ~~subbox~~sub-box \mathbf{X}^+
13 is less than or equal to the size of the ~~subbox~~sub-box \mathbf{X} .

1 30. (Original) The apparatus of claim 21, wherein the global optimizer is
2 configured to apply the Newton method to the John conditions.